All questions carry equal marks. *Total Marks 100*

1. LCM of two natural numbers is 495 and HCF is 5 . If the sum of the numbers is 100 , find out the difference between the two numbers.

   Solution: (LCM)(HCF) is equal to the product of the two numbers. So, the numbers are such that their product is (495)(5) and sum is 100. So, numbers are 45 and 55. **Ans. 10.**

2. $\frac{1}{\sqrt{9}-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-\sqrt{4}}=?$

   Solution: $\frac{1}{\sqrt{9}-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-\sqrt{4}}$
   $=\frac{\sqrt{9}+\sqrt{8}}{(\sqrt{9})^2-(\sqrt{8})^2}-\frac{\sqrt{8}+\sqrt{7}}{(\sqrt{8})^2-(\sqrt{7})^2}+\frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}-\frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6})^2-(\sqrt{5})^2}+\frac{\sqrt{5}+\sqrt{4}}{(\sqrt{5})^2-(\sqrt{4})^2}$
   $=\sqrt{9}+\sqrt{4}=5$ **Ans. 5.**

3. 7 men can complete a work in 12 days. They started the work and after 6 days, 1 of the men left. In how many days will the remaining work be completed by the remaining men?

   Solution: Work completed in 6 days is half the work. Each man completes the work - if working alone - in 84 days, so each man completes $\frac{1}{84}$ work in one day. So, six men complete $\frac{1}{14}$ work in one day. So, they will need 7 more days to complete half the work. **Ans. 7.**

4. A girl was lying on a bed and playing with her pearl necklace. The thread broke and some pearls fell down, if $1/3^{\text{rd}}$ fell to the ground, $1/5^{\text{th}}$ on the bed, $1/6^{\text{th}}$ in her hand, $1/10^{\text{th}}$ in her lap and the necklace still had 6 pearls in it. How many pearls were there in the necklace, originally?

   Solution: If there are x pearls, then we have $x=\frac{x}{3}+\frac{x}{5}+\frac{x}{6}+\frac{x}{10}+6$
   $\Rightarrow 30x=10x+6x+5x+3x+180 \Rightarrow x=30$ **Ans. 30.**

5. The product of the digits of a two digit number is one third of that number. If we add 18 to the original number, we get a number consisting of the same digits written in the reverse order. Find the original number.

   Solution: Suppose that the number is xy, i.e. unit place is y and tens place is x. So, we have $xy=\frac{10x+y}{3}$ and $10x+y+18=10y+x \Rightarrow y-x=2$. Replacing y by $x+2$ in the first condition, we get $3x^2+6x=10x+(x+2)=11x+2 \Rightarrow 3x^2-5x-2=0$ $\Rightarrow (x-2)(3x+1)=0 \Rightarrow x=2$ (since x is a natural number), so $y=4$. **Ans. 24.**

6. David gets on the elevator at the 11th floor of a building and rides up at the rate of 57 floors per minute. At the same time, Albert gets on an elevator at the 51st floor of the same building and rides down at the rate of 63 floors per minute. If they continue travelling at these rates, then at which floor will their paths cross?

   Solution: Suppose, initially David and Albert are n floors apart. In one minute, David will rise 57 floors and Albert will come down 63 floors, so they will be $n-(57+63)=n-120$ floors apart. So, every second, the distance between them reduces by two floors. Initially, they are 40 floors apart. So, they will need 20 seconds time to reduce that distance to zero. In 20 seconds, David will rise 19 floors, so they will meet at floor 30. **Ans. 30.**

7. The average temperature in degree Fahrenheit of the town in the first 4 days of the month was 58, the average for the second, third, fourth and fifth days was 60. If the temperatures of the first and fifth days were in the ratio 7:8 then what was the temperature on the 5th day?

   Solution: Sum of the temperatures of the first four days is $58 * 4 = 232$. Sum of temperatures of second, third, fourth and fifth day is $60 * 4 = 240$, so difference between fifth day's temperature and first day's temperature is 8. So, if the temperatures are $t_1 = 7x$ and $t_5 = 8x$, we have $8x - 7x = 8$. **Ans. 64.**

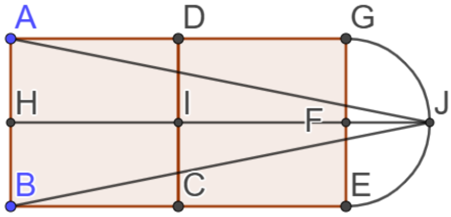
8. Amartya, Abhijeet, and Arun invest in a $5:6:7$ ratio with interest rates in a $2:3:4$ ratio. If Abhijeet earns 10 rupees more than Amartya, find total interest earned by all 3.

   Solution: Suppose the amounts invested are $5x, 6x, 7x$ rupees and interest rates are $2y, 3y, 4y$ p.c.p.a. So, interest earned by Amartya, Abhijeet and Arun are $\frac{10xy}{100}, \frac{18xy}{100}, \frac{28xy}{100}$ respectively.So, Abhijeet earns $\frac{18xy}{100} - \frac{10xy}{100} = \frac{8xy}{100} = 10$ more than Amartya. Total interest earned is $\frac{56xy}{100}$ which is 7 times $\frac{8xy}{100}$, so it is 70.
   Ans. 70.

9. The area of a right angled triangle is 24 sq . units and one side (not hypotenuse) of the triangle measures 6 units, find the perimeter of the triangle.

   Solution: Clearly, the other side forming the right angle is $\frac{2 \times 24}{6} = 8$. So, the hypotenuse is 10. **Ans. 24.**

10. In the figure, the squares $ABCD$ and $DCEG$ both have the same area of 64 Square units. EFG is a semicircle. The point F is the mid-point of the arc EFG. If the area of the shaded part is $4\pi + K$. Report K

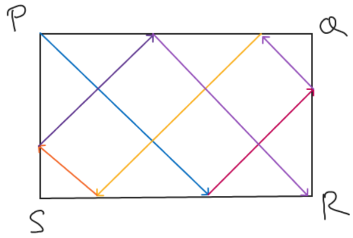
    Solution: The diagram is symmetric about the line HJ. Total area of the figure is two squares and one semicircle. Since area of one square is 64, each side is 8, so radius of the semicircle is 4. So, total area $= 64 + 64 + \frac{1}{2}\pi(16)$ and this is $=$ 2(shaded area) + area of $\triangle AJB$
    Area of $\triangle AJB = \frac{1}{2}(AB)(HJ) = \frac{1}{2}(8)(20) = 80$. So, we get $128 + 8\pi = 80 + 2($ shaded area $)$
    $\Rightarrow$ shaded area $= 24 + 4\pi$ **Ans. 24.**

11. Given that $A^4 = 75600 \times B$. If A and B are positive integers, find the smallest value of B. Report sum of the digits of B

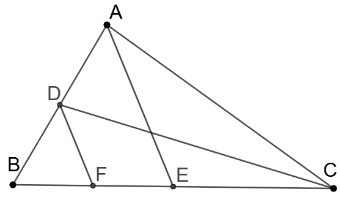
    Solution: $75600 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7 = 2^4 \times 3^3 \times 5^2 \times 7$
    So, smallest $B = 3 \times 5^2 \times 7^3 = 25725$ **Ans. 21.**

12. On a rectangular table $PQRS$ of 5 units long and 3 units wide, a ball is rolled from point P at an angle of $45°$ to PQ and bounces off SR at an angle of $45°$. The ball continues to bounce off the sides at $45°$ until it reaches R. How many times has the ball bounced?

Solution: As shown in the diagram, the ball bounces 6 times. **Ans. 6.**



13. In $\triangle ABC, D$ is the midpoint of AB, E is the midpoint of BC, F is the midpoint of BE and area of $\triangle DCF = 24$. Find the area of $\triangle ABC$.

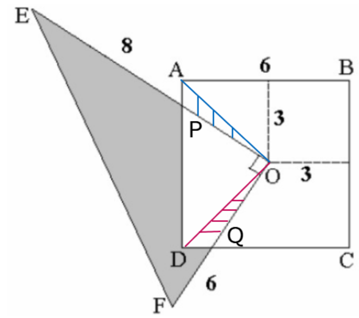


Solution: area of $\triangle DCF$: $\triangle DCB$ = 3 : 4 and area of $\triangle DCB : \triangle ABC = 1 : 2$ so area of $\triangle DCF : \triangle ABC = 3 : 8$ so area of $\triangle ABC = 64$ **Ans. 64.**

14. If the number $A1999311B$ is divisible by 72 , find the positive difference between A and B.

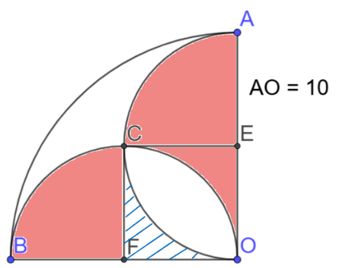
Solution: By divisibility test of 8, $11B$ must be divisible by 8, so $B = 2$. By divisibility test of 9, the sum of the digits should be divisible by 9, i.e. $A+1+9+9+9+3+1+1+2$ should be divisible by 9, so $A = 1$. **Ans. 1.**

15. In the figure, $ABCD$ is a 6×6 square with centre O . EOF is a right-angled triangle with $OE = 8$ and $OF = 6$. Find the area of the shaded region.



Solution: $\triangle AOP \cong \triangle DOQ$,
so area($\triangle POQ$) =area($\triangle AOD$) = 9, so area of the shaded region= area of($\triangle EOF$) $- 9 = 24 - 9 = 15$ **Ans. 15.**

16. Suppose OB and OA are diameters of the semicircles and $OB = OA = 10$, $m\angle AOB = 90$ is a right angle. A and B are two points on the circumference of circle of radius OA. Find the area of the shaded region.



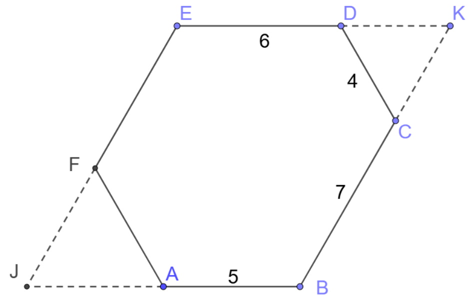
Solution: Area of the shaded region is 2(blue shaded area + quarter circle BFC) But quarter circle BFC has same area as the quarter circle ECO. So, shaded area= 2(area of $\square OECF$) = 2(25) **Ans. 50.**

17. If $x^3 = 1999$ and $y^2 = 1999$, where $x, y > 0$, find the number of integers between x and y .

Solution: $12^3 = 1728, 13^3 = 2197$, so $12 < x < 13$,
similarly $44^2 = 1936, 45^2 = 2025 \Rightarrow 44 < y < 45$, so integers between x and y are

$13, 14, 15, \cdots, 44$ **Ans. 32.**

18. All the angles of a hexagon are 120 degrees and four consecutive sides have lengths of 5, 7, 4, and 6 units. Find the sum of the lengths of the other two sides.



Solution: $\triangle DCK$ and $\triangle AFJ$ are equilateral triangles and $\square JEKB$ is a parallelogram. So, $EF + FA = EF + FJ = EJ = KB = KC + CB = DC + CB = 11$ **Ans. 11.**

19. When $24, 56, 104$ are divided by a positive integer k , they leave the same remainder. What is the greatest possible value of k ?

Solution: Suppose the same remainder is r. So, we have
$24 = k(n_1) + r$,
$56 = k(n_2) + r$,
$104 = k(n_3) + r$
$\Rightarrow 104 - 56 = k(n_3 - n_2), 104 - 24 = k(n_3 - n_1), 56 - 24 = k(n_2 - n_1)$, i.e. k is a common factor of $104 - 56, 104 - 24, 56 - 24$, i.e. $48, 80, 32$, so we need to find GCD of $32, 48, 80$. **Ans. 16.**

20. Find the 2025^{th} decimal digit when $\dfrac{1}{14}$ is expressed in decimal form.

Solution: $\dfrac{1}{14} = 0.0\overline{714285}$ Observe that $337 * 6 = 2022$, so digit at 2023^{th} position is 5. **Ans. 1.**

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
10	5	7	30	24	30	64	70	24	24	21	6	64	1	15	50	32	11	16	1